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Discretionary Policy and Multiple Equilibria in a New Keynesian Model

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Working Paper Series
2014-14

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First Version: September 2013

This Version: August 2014

Abstract

Focusing on linear-quadratic models with rational expectations, this paper extends the concept of discretionary equilibrium by allowing for linear non-Markovian strategies of the policy-maker and the other agents in the economy. Applying this concept to the standard New Keynesian framework, we show that a multitude of equilibria arise. Some equilibria have favorable consequences for welfare, resulting in outcomes superior even to those achieved under timeless-perspective commitment. Compared to traditional approaches to modeling credibility through trigger strategies, our approach has the desirable implication that small mistakes of the policy-maker have only small consequences for his reputation. Finally, we show that our equilibrium concept can provide an alternative explanation for the high degree of inflation persistence found in the data.

Keywords: New Keynesian model, multiple equilibria, discretionary equilibrium, inflation persistence, reputation.

JEL: E31, E52, E58, C61.

*I would like to thank Leo Kaas, Almuth Scholl, Karsten Wasiluk, Ivan Zyryanov, and seminar participants in Konstanz for many valuable comments and suggestions.

1 Introduction

This paper shows that discretionary policy-making may lead to multiple equilibria in linear-quadratic rational expectations models once the possibility is taken into account that the policy-maker and the other agents may respond to lagged endogenous variables that do not affect their current payoffs. To demonstrate this finding, we use the standard log-linearized New Keynesian model (see Clarida et al. (1999)) without endogenous payoff-relevant state variables such as capital. We focus on this model because it delivers a unique discretionary solution when only Markov perfect equilibria are considered (see Blake and Kirsanova (2012)).

While we believe our finding of multiple equilibria under discretionary policy-making to be of theoretical interest itself, we also prove that the additional equilibria that we identify can display arbitrary degrees of inertia. Thus they can be used as a potential explanation for the inflation persistence that is found in the data (see Nelson (1998) and Estrella and Fuhrer (2002)), but cannot be generated by a purely forward looking New Keynesian model without ad-hoc modifications like price indexation (see Christiano et al. (2005)) or rule-of-thumb price setting (see Galí and Gertler (1999)).[1] The position taken by this paper that inflation persistence is not a structural feature of the economy is supported by Benati (2008), who shows that inflation persistence is not invariant to changes in monetary-policy regimes.

Our equilibrium concept, which we label memory-dependent discretionary equilibrium (MDE), is a straightforward extension of the notion of discretionary equilibrium introduced by Oudiz and Sachs (1984) and Backus and Driffill (1986). The policy-maker maximizes welfare, taking its own future behavior and the behavior of the private sector as given. Due to the impossibility of committing to a specific future behavior,

[1]Rule-of-thumb price setting and indexation are theoretically unappealing because they cannot be obviously reconciled with firm optimizing behavior. Moreover, indexation is not in line with microeconomic evidence on firms' price setting (see Klenow and Malin (2010)). Alternative explanations for inflation persistence are due to Erceg and Levin (2003), who analyze the public sector's learning process about a time-varying monetary policy rule, Cogley and Sbordone (2008), who consider a New Keynesian model with trend inflation, Sheedy (2010), who studies time-dependent price-setting with a non-constant hazard function, and Niemann et al. (2013), who focus on the incentives to inflate away public debt.

each policy-maker can be viewed as an independent player. As a result, a new type of strategic complementarity arises in our framework between the actions of these players, which sets the stage for the occurrence of multiple equilibria:[2] The more vigorously future policy-makers respond to pay-off irrelevant variables, the stronger the optimal response of current policy-makers to these variables.

This dynamic complementarity can be explained in the following way. Suppose that future policy-makers respond to economic variables that are not payoff-relevant. Then the current choices of the private sector are influenced by these variables because, according to the New Keynesian Phillips curve, the private sector's decisions are affected by expectations about future economic variables and therefore depend on the decisions of future policy-makers. As a result, it is also optimal for current policy-makers to react to these payoff-irrelevant variables.

We show that some MDEs deliver a higher level of welfare than the standard discretionary solution in the canonical New Keynesian model (see Clarida et al. (1999), for example). While MDEs cannot mimic exactly the optimal commitment solution from a timeless perspective (see Woodford (1999)), which corresponds to the policy the policy-maker would have liked to commit to a long time ago, we show that MDEs can involve even higher welfare than timeless-perspective commitment.[3] Conversely, we prove that some MDEs have disastrous consequences for welfare. In these equilibria, the central bank is caught in an expectation trap where it has to confirm the private sector's expectations of large fluctuations of output and inflation in response to shocks.[4]

Our paper is related to several strands of the literature on multiple equilibria in the presence of discretionary policy-making. In non-linear models, the existence of multiple

[2]Cooper and John (1988) highlighted the important role of dynamic complementarities for equilibrium multiplicity.

[3]Dennis (2010) and Sauer (2010) demonstrate that a discretionary equilibrium can be superior to timeless-perspective commitment in the New Keynesian model.

[4]Our approach also has the potential to explain the Great Moderation, i.e. the observation that aggregate economic activity was particularly stable from the early 1980s until the 2007 financial crisis (see McConnell and Perez-Quiros (2000) for an early account of the Great Moderation). It is conceivable that during this period, the private sector and the central bank coordinated on a superior equilibrium.

equilibria when policy-makers act on a discretionary basis has been established by Albanesi et al. (2003) and King and Wolman (2004). Albanesi et al. (2003) construct Markov perfect equilibria where a sunspot variable may allow agents to coordinate on a particular equilibrium. King and Wolman (2004) point to the possibility of multiple equilibria and related sunspot equilibria in a New Keynesian model with two-period price setting.[5] Our paper differs from these contributions because we do not consider sunspot equilibria. We show that multiple equilibria arise when agents respond to endogenous pay-off irrelevant state variables.

In line with Blake and Kirsanova (2012), we assume that the policy-maker acts as a Stackelberg leader in each period. Because Eggertsson and Swanson (2008) show that simultaneous play of the private sector and the policy-maker leads to uniqueness in the framework proposed by King and Wolman (2004),[6] one might ask whether our findings are sensitive to this assumption. However, it is possible to prove that multiple MDEs in our framework would also arise if the policy-maker and the private sector moved simultaneously.[7]

Our paper is most closely related to Blake and Kirsanova (2012), who demonstrate the existence of multiple discretionary Markov equilibria in linear-quadratic models with rational expectations. More specifically, they point to the existence of endogenous state variables as a necessary precondition for the multiplicity of discretionary Markov equilibria. As the standard New Keynesian model adopted in our paper does not feature endogenous payoff-relevant state variables, their analysis implies that this model admits only a unique discretionary Markov equilibrium. The present paper complements Blake and Kirsanova's findings by showing that also payoff-irrelevant endogenous state variables can lead to multiple discretionary equilibria when we dispense of the assumption of Markovian strategies.

[5]Multiple discretionary equilibria in an open economy have been examined in Arseneau (2012). Armenter (2008) uses a one-period model to show that multiple equilibria may arise when welfare costs from inflation are bounded.

[6]Dotsey and Hornstein (2011) highlight that equilibrium is unique in King and Wolman (2004) if the central bank uses an interest rate as its instrument.

[7]A detailed analysis of a variant of our model where the central bank and the private sector move simultaneously is available upon request.

Finally, as our paper focuses on policies and private-sector choices that violate the Markov property, it is also related to works that examine how reputational mechanisms can overcome time-inconsistency problems.[8] For dynamic macroeconomic models, reputation-building can be modeled with the help of the sustainable equilibrium concept, which has been introduced by Chari and Kehoe (1990, 1993) and has been applied by various authors (Chang (1998), Ireland (1997), and Kurozumi (2005, 2008)).[9]

Our equilibrium concept differs from the notion of sustainable equilibrium in two respects. First, we consider only one-shot deviations by the policy-maker in each period rather than deviations which specify policies for all possible future histories.[10] One-shot deviations are of practical relevance because central banks typically determine merely current values for their instruments.[11,12]

Second, we restrict our attention to particular types of behaviors in equilibrium, namely those which depend on payoff-irrelevant state variables in a linear way. This assumption rules out trigger strategies, in particular. We consider this linearity requirement natural in a linear-quadratic model. It also has the desirable consequence that, unlike in the case with trigger strategies, small mistakes of the policy-maker would not have drastic consequences for economic aggregates and can never result in a complete break-down of reputation.[13]

We conclude the discussion of the differences of our equilibrium concept from the sustainable equilibrium concept by noting that the second difference, i.e. the focus

[8] See Barro and Gordon (1983) and Stokey (1989) for seminal contributions.

[9] Abreu et al. (1986, 1990) study reputation-building in infinitely repeated games by looking for fixed points of mappings of sets of values to sets of values.

[10] The one-shot deviation principle (see Blackwell (1965)) does not hold in our framework. This explains why it is possible that the standard discretionary equilibrium is the worst sustainable equilibrium in the standard New Keynesian model (see Kurozumi (2008)), while memory-dependent equilibria in our paper may involve lower welfare than the standard discretionary equilibrium.

[11] Forward guidance can be viewed as an exception. However, even under forward guidance, central banks are far from selecting values of their instruments for all possible future histories.

[12] Even if central banks could announce values of their instrument for every possible future contingency, it would still be interesting to analyze the case where this information is ignored by private agents. This is the case where only one-shot deviations are possible, which is examined in this paper.

[13] For example, in the classic paper by Barro and Gordon (1983) inflation expectations discontinuously jump to the inflation rate obtained under discretion whenever inflation differs from zero. More recently, trigger strategies of the private sector have been considered by Loisel (2008) and Levine et al. (2008).

on equilibria where strategies are of a particular linear form, tends to lead to smaller sets of equilibria when we utilize our equilibrium concept as opposed to sustainable equilibrium. By contrast, the first difference can be expected to result in larger sets of equilibria because MDEs are immune to smaller sets of possible deviations.

This paper is organized as follows. We lay out the model in the next section. In Section 3, we introduce a formal definition of the MDE concept. We characterize all MDEs, discuss the properties of an example MDE and derive our general results about inflation persistence in Section 4. In Section 5, we analyze the consequences that different MDEs have for welfare. Section 6 concludes.

2 Model

As has been mentioned before, we use the canonical New Keynesian model as our workhorse (see Clarida et al. (1999)). In each period $t = 0, 1, 2, ...$, the New Keynesian Phillips curve holds

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t + \xi_t, \tag{1}$$

where π_t is the inflation rate, y_t is the log output gap, β is the common discount factor ($0 < \beta < 1$), and κ a positive parameter. $\mathbb{E}_t[\pi_{t+1}]$ denotes the rational expectations about inflation in period $t+1$. Equation (1) can be derived from microeconomic foundations, as explained in detail in Woodford (2003, ch. 3, secs. 2.1 and 2.2).

The shocks ξ_t are i.i.d. from a normal distribution with zero mean and variance σ_ξ^2. They represent so-called cost-push or markup shocks, which can be microfounded by stochastic sales taxes or varying degrees of competition (see the discussion leading to Eq. (4.38) in Woodford (2003, ch. 6, sec. 4.5)).

The IS curve, which can be derived by log-linearizing the consumption Euler equation, is:

$$y_t = -\sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - \overline{r}) + \mathbb{E}_t[y_{t+1}] + \mu_t \tag{2}$$

Parameter σ ($\sigma > 0$) corresponds to the intertemporal elasticity of substitution, i_t is a short-term nominal interest rate, and $\overline{r}$ is the natural real rate of interest, which we assume to be constant and normalize to zero, for simplicity.[14] The shock μ_t is normally distributed with zero mean and variance σ_μ^2. Like the markup shocks, the demand shocks are independent across periods.

We would like to stress that we deliberately focus on the case where shocks are uncorrelated over time. As a result, any inflation persistence that will show up in our model will be generated by the memory-dependence of the central bank's and the other agents' actions and will not be caused by the persistence of shocks.

The per-period social loss function, which can also be derived from microeconomic foundations (see Woodford (2003, ch. 6, sec. 2.2)), is given by

$$l_t = \frac{1}{2}\pi_t^2 + \frac{a}{2}y_t^2, \tag{3}$$

where a ($a \geq 0$) is a parameter that gives the weight on output stabilization in the social loss function. Monetary policy is conducted by a central bank that shares society's preferences.

We extend the concept of discretionary equilibrium (see Oudiz and Sachs (1984) and Backus and Driffill (1986)) in a straightforward way to an equilibrium concept we label memory-dependent discretionary equilibrium (MDE). In each period t, the central bank chooses the preferred interest rate i_t, taking its own future policy and the public's expectations about the future output gap and inflation as given. Given the central bank's instrument i_t, the private sector chooses π_t and y_t in line with (1) and (2).[15] The major innovation in the present paper is that inflation expectations and the choices of the central bank and the private sector in each period t may depend on a state variable m_t called memory, which is a function of past inflation rates, output gaps as well as interest rates. Notably, these variables and therefore m_t do not enter the Phillips curve and the IS curve in period t and later periods. They also do not affect the central bank's current and future loss functions.

[14]Normalizing $\overline{r}$ to zero is not restrictive. It means that i_t in our model can be interpreted as the difference between the nominal interest rate and the natural real rate of interest or, alternatively, as the deviation of the nominal interest rate from its level in a zero-inflation steady state.

[15]This particular timing assumption that the central bank moves first and the private sector moves second, after observing the central bank's choice, is in line with Blake and Kirsanova (2012) (see their Assumption 2 on p. 1330).

More specifically, we define memory m_t recursively in the following way:

$$m_t = \phi_m m_{t-1} + \phi_\pi \pi_{t-1} + \phi_y y_{t-1} + \phi_i i_{t-1} \qquad \text{for } t \geq 1, \tag{4}$$

where the ϕ's are coefficients left to be determined. We note that m_t is pre-determined at t. The initial value m_0 is exogenously given. While at this point m_t has no natural interpretation and is simply a variable that aggregates the lags of the economic variables in our model, we will see in the course of our analysis that m_t can be identified with the past expectations of current inflation.

Our specification of m_t in Equation (4) satisfies the following three properties. First, it represents a comparably small deviation from the standard discretionary equilibrium by allowing the economy to be affected by only one additional state variable. This simplicity makes it possible to derive analytical results. We will see that already this minor modification of the concept of discretionary equilibrium leads to a host of additional equilibria. Second, the additional state variable m_t is allowed to depend on one-period lags of all endogenous aggregate variables π_t, y_t, and i_t. Third, these variables affect m_t linearly, which appears to be natural in a linear-quadratic model. As a consequence, the equilibrium choices of the central bank and the public will be linear functions of the current shocks and lagged endogenous economic variables.

We would like to emphasize that we require expectations to be rational. Thus, in an equilibrium where the private sector makes its choices dependent on memory because it expects the central bank to respond to memory in the future, the central bank will in fact find it optimal to respond to m_t in the way expected by the agents in the economy. Our equilibrium concept, MDE, will be defined formally in the next section. We will see that the class of MDEs contains the traditional discretionary solution, which constitutes a Markov perfect equilibrium, as a special case.

3 Equilibrium Concept

For a formal definition of MDEs, a few preliminary steps are necessary. In each period t, we introduce the vector of state variables as

$$s_t = (m_t, \xi_t, \mu_t). \tag{5}$$

We assume that, in equilibrium, the private sector's choices of π_t and y_t will be linear functions of these state variables. Thus, we can write

$$\pi_t = \psi_m^\pi m_t + \psi_\xi^\pi \xi_t + \psi_\mu^\pi \mu_t, \tag{6}$$

$$y_t = \psi_m^y m_t + \psi_\xi^y \xi_t + \psi_\mu^y \mu_t, \tag{7}$$

where the ψ's are coefficients left to be determined. We follow the previous literature (Oudiz and Sachs (1984), Backus and Driffill (1986), and Blake and Kirsanova (2012)) in assuming stationary private-sector behavior and central bank policies. Thus the ψ's are independent of time.

Equations (6) and (7) imply that the expectations about inflation and the output gap can be written as

$$\mathbb{E}_t[\pi_{t+1}] = \psi_m^\pi m_{t+1} = \psi_m^\pi \left(\phi_m m_t + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t\right), \tag{8}$$

$$\mathbb{E}_t[y_{t+1}] = \psi_m^y m_{t+1} = \psi_m^y \left(\phi_m m_t + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t\right), \tag{9}$$

where we have taken into account $\mathbb{E}_t[\xi_{t+1}] = \mathbb{E}_t[\mu_{t+1}] = 0$ and used (4) to replace m_{t+1}.

With the help of (8) and (9), we can combine (1) and (2) to derive equations specifying the private-sector responses to the central bank's policy i_t. We use $\mathcal{P}(i_t, s_t)$ for the private sector's choice of inflation and $\mathcal{Y}(i_t, s_t)$ for the respective choice of output gap.[16] These expressions are stated formally in Appendix A.

[16] At this point, one might wonder about the difference between $\mathcal{P}(i_t, s_t)$ and the expression for π_t described in (6), which does not depend on i_t. The latter expression describes the equilibrium behavior of inflation under the assumption that the central bank chooses its equilibrium policy. Using the central bank's equilibrium choice of i_t, which is a function of s_t only, to substitute for i_t in $\mathcal{P}(i_t, s_t)$ yields an expression for π_t that depends only on the state s_t like the expression for π_t in (6).

Given the private sector responses $\mathcal{P}(i_t, s_t)$ and $\mathcal{Y}(i_t, s_t)$, the central bank's optimal behavior is described by the following Bellman equation:

$$\begin{aligned} V(s_t) = \min_{i_t} &\left\{ \frac{1}{2} \left(\mathcal{P}(i_t, s_t)\right)^2 + \frac{a}{2} \left(\mathcal{Y}(i_t, s_t)\right)^2 + \beta \mathbb{E}_t V(s_{t+1}) \right\} \\ s.t.\ m_{t+1} &= \phi_m m_t + \phi_\pi \mathcal{P}(i_t, s_t) + \phi_y \mathcal{Y}(i_t, s_t) + \phi_i i_t. \end{aligned} \tag{10}$$

In Appendix B, we draw on this Bellman equation to derive the following condition for optimal central bank behavior:

$$\begin{aligned} &\left(\left(1 - \psi_m^\pi \phi_i - \frac{\psi_m^y \phi_i}{\sigma} \right) \kappa + \beta \psi_m^\pi \left(\phi_y - \frac{\phi_i}{\sigma} \right) \right) \pi_t \\ &+ a \left(1 - \psi_m^\pi \phi_i - \beta \psi_m^\pi \phi_\pi - \frac{\psi_m^y \phi_i}{\sigma} \right) y_t \\ =&\beta \phi_m \left(\kappa \mathbb{E}_t[\pi_{t+1}] + a \mathbb{E}_t[y_{t+1}] \right) \end{aligned} \tag{11}$$

This equation is somewhat unwieldy and difficult to interpret. In contrast with the optimality condition for the standard discretionary equilibrium, $0 = \kappa \pi_t + a y_t$ (see Clarida et al. (1999, Eq. (3.3), p. 1672)), it depends on expectations about future inflation and the output gap. This is a consequence of the observation that monetary policy in period t potentially affects economic variables in the next period $t+1$ because these may depend on m_{t+1} and thereby on π_t, y_t, and i_t. We will see in the next section that condition (11) contains the optimality condition for the standard discretionary equilibrium as a special case.

We can now define an MDE for the canonical New Keynesian model:

Definition 1. *For a given initial value $m_0 \in \mathbb{R}$, an MDE is a mapping from all possible paths of shocks $\{\xi_t\}_{t=0}^\infty$ and $\{\mu_t\}_{t=0}^\infty$ to paths of inflation, the output gap and the interest rate, i.e. $\{\pi_t\}_{t=0}^\infty$, $\{y_t\}_{t=0}^\infty$, $\{i_t\}_{t=0}^\infty$, for which the following two properties hold.*

1. *Equations (1), (2), (4), (6), (7), and (11) hold for all periods $t = 0, 1, \ldots$.*

2. *For all possible initial states μ_0 and ξ_0, $\lim_{t\to\infty} \mathbb{E}_0[\pi_t] = 0$ and $\lim_{t\to\infty} \mathbb{E}_0[y_t] = 0$.*[17]

[17]In the knife-edge case $m_0 = 0$, we postulate $\lim_{t\to\infty} \mathbb{E}_0[\pi_t] = 0$ and $\lim_{t\to\infty} \mathbb{E}_0[y_t] = 0$ for all admissible values of m_0, μ_0, and ξ_0.

We note that this definition implies that each MDE can be characterized by a tuple of coefficients $(\psi_m^\pi, \psi_\xi^\pi, \psi_\mu^\pi, \psi_m^y, \psi_\xi^y, \psi_\mu^y, \phi_m, \phi_\pi, \phi_y, \phi_i)$.

While the first condition in Definition 1 follows from our previous arguments, the second condition is imposed to rule out explosive equilibria. Ruling out explosive solutions is in the tradition of Blanchard and Kahn (1980) and ensures that a first-order approximation of the private sector's behavior is adequate. Sims (2002) argues that this approach is too restrictive in linear-quadratic models with rational expectations and proposes to exclude only the solutions growing at a rate higher than $1/\sqrt{\beta}$, as only these would violate transversality conditions. Imposing this looser condition would lead to an even larger set of equilibria and thereby would strengthen our findings.

4 Characterization of MDEs

Before characterizing the entire set of MDEs, we confirm that the standard discretionary equilibrium found in the literature (Clarida et al., 1999) is an MDE. For this purpose, we observe that $\psi_m^\pi = \psi_m^y = 0$ entails $\mathbb{E}_t[\pi_{t+1}] = \mathbb{E}_t[y_{t+1}] = 0$ (see (8) and (9)), which in turn implies that (11) becomes $0 = \kappa\pi_t + ay_t$. This is the standard condition obtained in the literature for the discretionary equilibrium (see Clarida et al. (1999, Eq. (3.3), p. 1672)).[18] Inserting $\mathbb{E}_t[\pi_{t+1}] = 0$ into the Phillips curve (1) enables us to compute expressions for inflation and the output gap. Hence, we immediately obtain the following lemma:

Lemma 1. *The standard discretionary equilibrium, where*

$$\pi_t = \frac{a}{a+\kappa^2}\xi_t, \tag{12}$$

$$y_t = -\frac{\kappa}{a+\kappa^2}\xi_t, \tag{13}$$

is an MDE.

[18] Recall that we focus on shocks that are not persistent. Thus, in contrast with Clarida et al. (1999), the expectations about future output and inflation are unaffected by current shocks, i.e. $\mathbb{E}_t[\pi_{t+1}] = \mathbb{E}_t[y_{t+1}] = 0$.

As is well-known, in the standard discretionary equilibrium the central bank can fully stabilize demand shocks μ_t, which therefore do not affect inflation and output. In the presence of markup shocks ξ_t, an influence of these shocks on inflation can typically not be avoided. How strongly these shocks affect inflation depends on a, the central bank's weight on output stabilization in its loss function. According to (13), output is less affected by markup shocks when a is high.

Obviously, Lemma 1 establishes the existence of an MDE in our economy. It is unclear as yet whether additional MDEs exist. The following Lemma introduces an invariance property, which will be useful for the characterization of additional MDEs:

Lemma 2. *Suppose that, for a given initial value of m_0, an MDE can be characterized by the tuple $(\psi^\pi_m, \psi^\pi_\xi, \psi^\pi_\mu, \psi^y_m, \psi^y_\xi, \psi^y_\mu, \phi_m, \phi_\pi, \phi_y, \phi_i)$. Then the same MDE can also be characterized by $(\hat{\psi}^\pi_m, \hat{\psi}^\pi_\xi, \hat{\psi}^\pi_\mu, \hat{\psi}^y_m, \hat{\psi}^y_\xi, \hat{\psi}^y_\mu, \hat{\phi}_m, \hat{\phi}_\pi, \hat{\phi}_y, \hat{\phi}_i) = \left(\lambda\psi^\pi_m, \psi^\pi_\xi, \psi^\pi_\mu, \lambda\psi^y_m, \psi^y_\xi, \psi^y_\mu, \phi_m, \frac{1}{\lambda}\phi_\pi, \frac{1}{\lambda}\phi_y, \frac{1}{\lambda}\phi_i\right)$ $\forall\lambda \neq 0$ for the initial value $\hat{m}_0 = m_0/\lambda$.*

The proof is straightforward. Taking (4), (6), and (7) into account reveals that the transformation specified in the lemma scales m_t by a factor $1/\lambda$ but leaves the paths of all economically meaningful variables $\{\pi_t\}^\infty_{t=0}$, $\{y_t\}^\infty_{t=0}$, and $\{i_t\}^\infty_{t=0}$ unchanged.

It is immediate to see that the standard discretionary equilibrium described in Lemma 1 fulfills this invariance property because ψ^π_m, ψ^y_m, ϕ_π, ϕ_y, and ϕ_i are all zero in this case. Lemma 2 has the consequence that we will be able to normalize the coefficients $(\psi^\pi_m, \psi^\pi_\xi, \psi^\pi_\mu, \psi^y_m, \psi^y_\xi, \psi^y_\mu, \phi_m, \phi_\pi, \phi_y, \phi_i)$ in a convenient way.

The following proposition, which is proved in Appendix C, characterizes all MDEs different from the one described in Lemma 1:

Proposition 1. *All MDEs that differ from the equilibrium characterized in Lemma 1 can be constructed in the following way.*

1. *Normalize $\psi^\pi_m = 1$.*

2. *Pick arbitrary $(\psi^y_m, \phi_m, \phi_i)$ with $\phi_m \neq 0$ and $\psi^y_m \in \left(\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}\right) \setminus \mathbb{L}$.*

3. *For given* $(\psi_m^y, \phi_m, \phi_i)$*, the unique solutions for* ψ_ξ^π*,* ψ_ξ^y*,* ϕ_π*,* ϕ_y*,* ψ_μ^π*, and* ψ_μ^y *are determined by Equations (28), (31), (33), (34), (37), and (38), specified in Appendix C.*

The set $\mathbb{L}$ contains at most four points and is defined in Appendix C.[19]

In line with Proposition 1, we can define $\mathbb{A} := \big(\big(\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}\big) \setminus \mathbb{L}\big) \times (\mathbb{R} \setminus \{0\}) \times \mathbb{R}$ as the set of admissible combinations of $(\psi_m^y, \phi_m, \phi_i)$. Each of these combinations leads to a unique solution for ϕ_π, ϕ_y, ψ_μ^π, ψ_μ^y, ψ_ξ^π, and ψ_ξ^y. Hence we will use in the following the terminology that a particular combination $(\psi_m^y, \phi_m, \phi_i) \in \mathbb{A}$ characterizes an equilibrium.

In line with Lemma 2, it has been possible to normalize ψ_m^π to one in Proposition 1. For this normalization, m_t equals past inflation expectations:

$$m_t = \mathbb{E}_{t-1}[\pi_t], \tag{14}$$

which is easily verified from (6). Hence lagged inflation expectations can be interpreted as a state variable in our framework.[20] It is instructive to look at the dynamics of inflation expectations. Using (8) and (14), we obtain

$$\mathbb{E}_t[\pi_{t+1}] = \phi_m \mathbb{E}_{t-1}[\pi_t] + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t. \tag{15}$$

Note that π_t, y_t, and i_t are functions of the exogenous shocks and $m_t = \mathbb{E}_{t-1}[\pi_t]$ in equilibrium. Consequently, we can express $\mathbb{E}_t[\pi_{t+1}]$ solely as a function of the exogenous shocks ξ_t and μ_t as well as lagged inflation expectations $\mathbb{E}_{t-1}[\pi_t]$. As current expectations about future inflation depend on past inflation expectations, our theory can be thought of as providing a role for adaptive expectations formation, thereby reconciling adaptive and rational expectations.

At this point, we demonstrate the existence of additional MDEs and illustrate the typical properties of these equilibria by analyzing the impulse responses of output and

[19] For $\beta = 0.99$, $\kappa = 0.3$, $\sigma = 1$, and $a = 0.03$, i.e. the parameter values that we will select for Figure 3, the intersection of $\mathbb{L}$ and $\big((1-\beta)/\kappa, (1+\beta)/\kappa\big)$ contains a single point.

[20] In line with (7), m_t is also proportional to past expectations about current output. If we used a different normalization, we could ensure $m_t = \mathbb{E}_{t-1}[y_t]$.

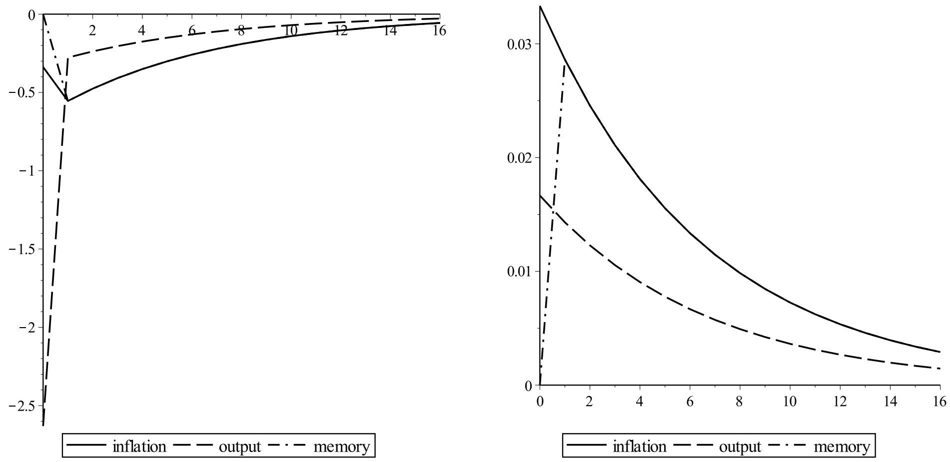

Figure 1: Impulse responses of inflation, the output gap, and "memory" for cost-push shocks (left-hand side) and demand shocks (right-hand side).

inflation for a particular example. For this purpose, we pick specific values for the exogenous parameters in our model. We adopt the standard values selected by Clarida et al. (2000). For quarterly data, we choose $\beta = 0.99$ and $\kappa = 0.3$. We set σ, which corresponds to the intertemporal elasticity of substitution in the household's CRRA utility function of the underlying microeconomic model, to one.[21] A derivation of the social loss function from the utility function of the representative household yields an expression κ/θ for the weight a, where θ is the elasticity of substitution in the Dixit-Stiglitz index of aggregate demand (see Woodford (2003, ch. 6, sec. 2)). The markup under monopolistic competition is $1/(\theta - 1)$ over marginal costs. A plausible markup of 10% leads to a value of $\theta = 11$ and thus $a = \kappa/\theta \approx 0.03$.

For these parameter values, one example MDE is given by $\phi_i = 0.1$, $\phi_\pi = -2.522$, $\phi_y = 0.603$, $\phi_m = 3$, $\psi_m^\pi = 1$, $\psi_\xi^\pi = -0.338$, $\psi_\mu^\pi = 0.033$, $\psi_m^y = 0.5$, $\psi_\xi^y = -2.631$, and $\psi_\mu^y = 0.017$.[22] Figure 1 displays the impulse responses of inflation (solid line), the output gap (dashed line), and memory (dashed-dotted line) for markup shocks on the left-hand side and demand shocks on the right-hand side.[23] First, it is noteworthy that

[21]Compare Woodford (2003, ch. 4, sec. 1.2, eq. (1.7)) for the case where the share of private expenditure in total aggregate demand is one.

[22]One might wonder whether $\phi_m > 1$ does not lead to explosive dynamics for m_t and thereby for π_t and y_t. This, however, is not the case because π_t, y_t, and i_t are also functions of m_t in equilibrium (see (4)).

[23]The impulse responses of the interest rate, which are not displayed, feature a spike at period 0 and a comparably smooth path for later periods.

inflation shows a persistent response to both types of shocks, although these shock only have a direct effect on the economy in period 0. Second, we observe that memory and inflation are identical from period 1 on. This reflects our earlier finding that memory in period t corresponds to the one-period lag of inflation expectations, i.e. $m_t = \mathbb{E}_{t-1}[\pi_t]$. Finally, the figure shows that, both under markup shocks and under demand shocks, the output gap and inflation have the same sign for all periods $t \geq 1$.[24] This is in line with the finding, provided in Appendix C, that ψ_m^π and ψ_m^y have the same sign in all MDEs, irrespective of the normalization of ψ_m^π.

Our example has demonstrated the existence of multiple equilibria in our framework. This raises the important question of why such multiplicity arises. The potential for multiple equilibria can be seen in light of strategic complementarities (see Cooper and John (1988)). Note that the central bank in each period can only choose the current interest rate i_t but has to take its own behavior in all other periods as given. Thus, we can interpret an MDE as a strategic interaction of infinitely many central banks, each of them active in a particular period t. Strategic complementarities arise because a central bank in period t finds it optimal to respond to memory m_t more strongly, the more strongly the central banks in the future respond to memory.

These strategic complementarities can be understood more clearly with the help of Figure 2. In this figure, a solid arrow is used to describe the fact that one variable influences the other. Hence the arrow between the upper-right box and the box below stands for the presumption that the future central bank will make its policy dependent on future memory m_{t+1}. In this case, the choices of the private sector, moving after the central bank, will also be affected by m_{t+1}. Thus the three boxes on the right provide an explanation (illustrated by the dashed arrow) for the fact that expectations in t about inflation and output in $t+1$ are affected by period-t expectations about m_{t+1} (see the two boxes in the middle, connected by the solid arrow pointed at by the dashed arrow).

[24]In fact, this observation is also true for $t \geq 0$ in this example. However, this pattern does not hold for all MDEs.

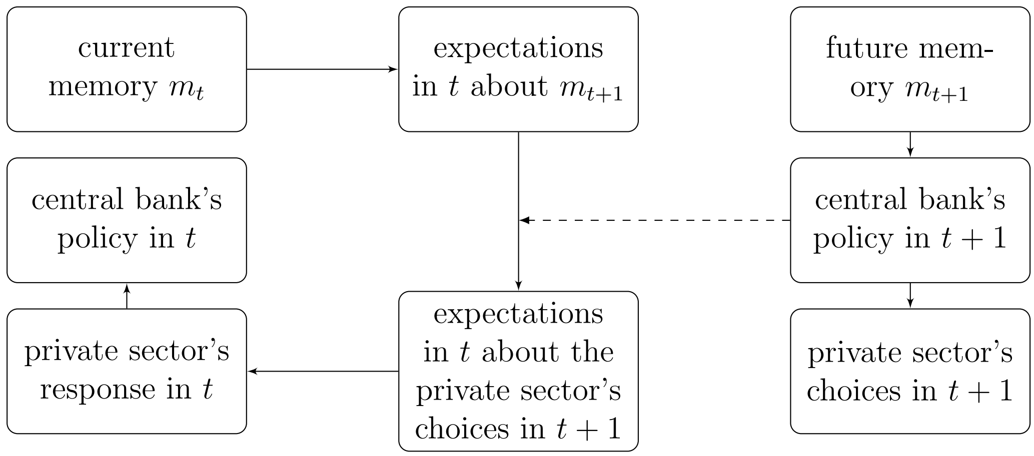

Figure 2: A diagram illustrating the dynamic complementarities leading to multiple equilibria. Solid arrows indicate that a variable affects the other. The dashed arrow indicates that the relationships illustrated by the three boxes on the right provide an explanation for the respective solid line.

We can now easily interpret the remainder of the figure, starting from the upper-left corner and moving in the directions indicated by the solid arrows. Due to the recursive definition of memory (see (4)), current memory influences expectations about future memory, which in turn affect expectations in t about inflation and output in $t+1$, depicted by the large box at the bottom. The Phillips curve (1) and the IS curve (2) imply that current output and inflation depend on these expectations (see the bottom-left arrow, pointing leftwards). Finally, the central bank anticipates that the private sector's response will depend on memory m_t in period t. As a result, its policy depends on m_t as well (see the short arrow on the left, pointing upwards). To sum up, when the central bank is expected to respond to memory in the future, then it is also optimal for the central bank to respond to memory currently.

We note that our diagram suggests a crucial feature of additional MDEs: Future memory, m_{t+1}, has to be a function of current memory, m_t. Otherwise the link described by the upper-left arrow would be absent and hence there would be no strategic complementarities between the actions of policy-makers operating in different periods. In fact, this finding holds generally, as is stated in the following corollary, which is a consequence of Lemma 1 and Proposition 1:[25]

[25]It is straightforward to see that the standard equilibrium can be represented by parameter con-

Corollary 1. *There is no MDE with $\phi_m = 0$ that differs from the standard equilibrium.*

Put differently, no MDE exists where the agents' choices depend only on one-period lags of inflation, the output gap, and the interest rate.[26]

In Lemma 2, we have already observed that different sets of coefficients $(\psi_m^\pi, \psi_\xi^\pi, \psi_\mu^\pi, \psi_m^y, \psi_\xi^y, \psi_\mu^y, \phi_m, \phi_\pi, \phi_y, \phi_i)$ may describe the same equilibrium. This raises the question of whether the different sets of coefficients whose construction is outlined in Proposition 1 actually describe different equilibria. To answer this question, we introduce some additional terminology. We say that two admissible combinations $(\psi_m^y, \phi_m, \phi_i), (\hat{\psi}_m^y, \hat{\phi}_m, \hat{\phi}_i) \in \mathbb{A}$ characterize the same equilibrium if, for all $m_0 \in \mathbb{R}$, there is an $\hat{m}_0 \in \mathbb{R}$ such that, for both combinations, the procedure outlined in Proposition 1 leads to the same mapping from exogenous states $\{\mu_t\}_{t=0}^\infty$, $\{\xi_t\}_{t=0}^\infty$ to paths of endogenous variables $\{\pi_t\}_{t=0}^\infty$, $\{y_t\}_{t=0}^\infty$, $\{i_t\}_{t=0}^\infty$ when the initial value of the memory variable is m_0 in the first case and $\hat{m}_0$ in the second case. Two admissible combinations are said to characterize different equilibria otherwise.

After these steps, we can prove that different combinations of coefficients described in Proposition 1 indeed correspond to different MDEs:

Proposition 2. *Any two different $(\psi_m^y, \phi_m, \phi_i), (\hat{\psi}_m^y, \hat{\phi}_m, \hat{\phi}_i) \in \mathbb{A}$ characterize different equilibria.*

The proof is given in Appendix D. Recall that Proposition 1 implies that ϕ_i can be any real number, ϕ_m can be any non-zero real number, and ψ_m^y can be freely chosen from $\left(\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}\right) \setminus \mathbb{L}$. Proposition 2 proves that each combination of these variables delivers a different equilibrium. Therefore these propositions imply jointly that a continuum of MDEs exist in our model.

Blake et al. (2013) examine whether the equilibrium multiplicity found in Blake and Kirsanova (2012) can be eliminated by delegating monetary policy to an authority with different preferences than society. In a similar vein, one might wonder whether

stellations with $\phi_m = 0$.

[26] We note that the alternative timing assumption that the central bank and the private sector move simultaneously in each period allows for such equilibria.

delegation to a completely conservative central banker would do away with the additional MDEs in our paper. This, however, is not the case. We would like to stress that Proposition 2 holds for all admissible parameter constellations, including those with $a = 0$. Hence, the delegation of monetary policy to a conservative central bank cannot implement a unique equilibrium.

We have already highlighted in the Introduction that MDEs can potentially explain inflation persistence. The next proposition, which is proved in Appendix E, shows exactly this:

Proposition 3. *In all MDEs, the impulse response of inflation to both shocks ξ_0 and μ_0 for $m_0 = 0$ declines exponentially from period 1 on. For arbitrary $\rho \in [0, 1)$, it is possible to construct an MDE for which the impulse responses of inflation to demand shocks and cost-push shocks decline at this rate, i.e. for which π_t is proportional to ρ^t for $t = 1, 2, 3, \ldots$.*

Loosely speaking, the proposition states that MDEs of the canonical New Keynesian model are compatible with arbitrary degrees of inflation persistence.

5 Welfare Analysis

In the following, we analyze the consequences that different MDEs have for welfare. More specifically, we compare the additional MDEs characterized in Proposition 1 to the standard discretionary equilibrium in Lemma 1 and to timeless-perspective commitment. We use the calibration described earlier. In addition, we assume that the variances of cost-push shocks and of demand shocks are identical. We stress that all our results are not sensitive to this assumption.[27]

At this point, a few words are in order regarding our welfare measure. We use the criterion traditionally employed in the literature, namely the unconditional expectation

[27] At first glance, a larger value of the variance of demand shocks appears to make non-standard MDEs worse compared to the optimal commitment solution, which is considered in Proposition 4. This follows from the fact that, in non-standard MDEs, inflation and output are influenced by demand shocks. However, even for high variances of demand shocks, the statement of Proposition 4 continues to hold. In these cases, the MDEs with the highest welfare levels guarantee that demand shocks have a negligible impact on inflation and output.

of social losses (see the contributions in Taylor (1999), for example). Dennis (2010) proposes social losses conditional on the predetermined state variables and integrated over auxiliary state variables as an alternative measure. In our case, the auxiliary state variables would be m_0 for MDEs and y_{-1} for timeless-perspective commitment. More specifically, Dennis (2010) argues that unconditional losses have the disadvantage of disregarding transition dynamics. This point is less relevant in our case because we merely compare different stationary equilibria and do not consider policy changes. Moreover, one difficulty of applying his approach to our paper would be that it requires the calculation of the distribution of the auxiliary state variables m_0 or y_{-1} respectively, conditional on the states ξ_0 and μ_0. These conditional distributions would be different for both equilibria under comparison when we compare two different MDEs or a specific MDE with timeless-perspective commitment. Hence one would have to make a somewhat arbitrary decision on which of the two possible distributions to select for the welfare comparison.

In Appendix F, we compute analytical expressions for social welfare, measured by the unconditional expectation of social losses, for commitment from a timeless perspective as well as for arbitrary MDEs, which include the standard discretionary equilibrium as a special case. Using a grid search, we identify the MDE with the lowest social losses, which gives us the following result:

Proposition 4. *MDEs can display lower levels of unconditional social losses than the standard discretionary equilibrium or the timeless-perspective commitment equilibrium.*

The proposition shows that some MDEs allow for welfare gains through a smooth reputational mechanism, where we use the term "smooth" to describe the fact that small deviations of actual policies from expected policies do not affect future outcomes strongly. We consider this a plausible feature of our approach compared to a scenario where the public adopts trigger strategies because small errors in monetary policy-making or minor misperceptions of the central bank's strategy by the private sector are unavoidable for real-life central banks and should not have severe consequences for a central bank's reputation. Importantly, Proposition 4 claims that MDEs can be socially preferable even to timeless-perspective commitment.[28]

At first glance, it might appear surprising that equilibria with lower unconditional losses than the timeless-perspective commitment solution can exist. At this point, it is important to remember that commitment from a timeless perspective is the solution that a Ramsey planner would have committed to a long time ago. This solution is distinct from the stationary policy minimizing unconditional losses. This can be seen by drawing an analogy. Consider the social planner's solution for the textbook neoclassical growth model. Assume, for simplicity, a constant efficiency of labor, a constant population, as well as no disutility from labor.[29] The balanced growth path implied by the social planner's solution, which is the analogue of the timeless-perspective commitment solution in our model, is not the stationary allocation with the highest utility of the representative agent, which could be achieved by the golden-rule level of the capital stock.

It is instructive to examine the socially best MDE more closely. For this purpose we present the impulse responses of inflation and the output gap to markup shocks for the socially optimal MDE and the optimal commitment solution from a timeless perspective (see Figure 3). The impulse responses of inflation and the output gap are virtually indistinguishable. In particular, we note that both impulse responses of inflation display an overshooting of the inflation rate to cost-push shocks. This overshooting is socially desirable in the absence of persistent shocks because a below-average inflation in the future can counteract the current inflationary pressure exerted by a positive markup-shock.

It is well-known that under optimal timeless-perspective commitment, inflation and output are completely unresponsive to demand shocks. While this property does not hold exactly in the socially best MDE, the response to demand shocks is very small. Our

[28]This observation is related to Dennis (2010) and Sauer (2010), who prove that discretionary equilibria can be superior to timeless-perspective commitment in some variants of the New Keynesian model. However, as has been detailed before, Dennis (2010) uses a different welfare measure than the one employed here.

[29]See Acemoglu (2009, Sec. 5.9) for an exposition.

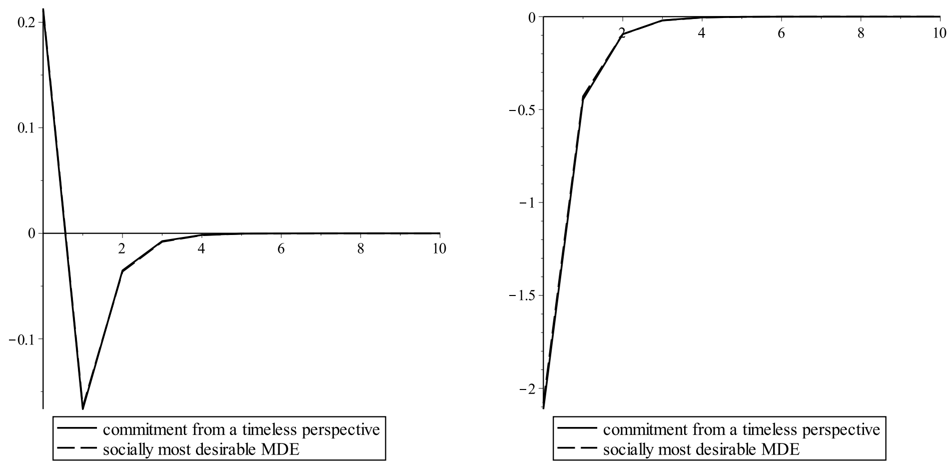

Figure 3: Impulse response of inflation (left-hand side) and the output gap (right-hand side) to a cost-push shock.

simulations reveal that the response is the smaller, the higher the variance of demand shocks. This observation explains why even for high variances of demand shocks, the socially best MDE can be superior to the optimal policy under commitment from a timeless perspective.

While the MDE considered in Proposition 4 has particularly benign consequences for welfare, there are also MDEs with arbitrarily high social losses. This is stated in our last proposition, which is proved in Appendix G:

Proposition 5. *Suppose that at least one of the shocks has strictly positive variance, i.e. either $\sigma_\mu^2 > 0$ or $\sigma_\xi^2 > 0$ or both. Then the unconditional expectation of social losses implied by MDEs can be arbitrarily high.*

Intuitively, this finding is the consequence of our result that shocks can have arbitrarily high persistence in our framework. Therefore even shocks with a small but positive variance can have severe consequence for the unconditional variances of inflation and output.

It is interesting to contrast our finding of potentially high welfare losses with the observation in Kurozumi (2008) that the worst sustainable equilibrium is the standard discretionary equilibrium. According to the sustainable equilibrium concept, the policy-maker can pick in each period a complete history-contingent plan for its instrument in the future, subject only to the constraint that he will not find it profitable to deviate from this plan in the future. Hence, the policy-maker can always deviate to the path implementing the standard discretionary equilibrium. In our paper, the central bank can only select its instrument in the current period and has to take its own future behavior as given.[30] As a result, the central bank can be caught in an expectation trap, where it has to confirm private-sector expectations that correspond to an equilibrium with a low level of welfare.

6 Conclusions

In this paper, we have shown that the canonical New Keynesian model admits a continuum of discretionary equilibria if we allow private-sector actions as well as the central bank's choices to depend on past endogenous states that are not payoff-relevant currently. We have demonstrated that the additional state variable showing up in our model is identical to past inflation expectations. Interestingly, the additional discretionary equilibria identified in our paper have the potential to explain the degree of inflation persistence found in the data. While some memory-dependent discretionary equilibria have disastrous consequences for welfare, others are superior to the standard discretionary solution and even to timeless-perspective optimal commitment.

The existence of a continuum of equilibria in our framework raises two questions: First, does our equilibrium concept lead to any restrictions on the dynamics of the system in addition to those implied by the Phillips curve and the IS curve? Second, is it possible to refine our notion of equilibrium in order to reduce the number of equilibria and thereby to make sharper predictions?

The first question is easy to answer. The first-order condition for the optimal behavior of the policy-maker, i.e. Eq. (23), involves additional restrictions on the dynamics of the system. These restrictions constrain the set of admissible coefficients describing

[30] As has been mentioned before, the one-shot deviation principle fails to hold in our framework.

the responses of the policy-maker and the private agents. The second question is more challenging. Nevertheless we would like to offer a few tentative thoughts. First, one might be tempted to prefer discretionary policy-making under the Markov assumption on the grounds that this approach yields a unique equilibrium or at least significantly less equilibria. However, even if the Markov assumption reduces the set of equilibria effectively, it is unclear why this restriction should be the economically most relevant one. Thus our paper highlights that the assumption of Markovian strategies is by no means innocuous. Second, previous studies on reputation-building by central banks are also plagued by multiple equilibria. For example, in the seminal paper by Barro and Gordon (1983) a continuum of what the authors call enforceable rules exist. The authors therefore consider the case where the policy-maker implements the enforceable rule that guarantees minimal social losses. In our context, an application of this principle would lead to the adoption of the socially best MDE, which was discussed in Section 4.[31] Finally, it would be attractive to develop alternative equilibrium refinements. The approach pursued by Dennis and Kirsanova (2013), who develop a concept of learnability for discretionary sunspot equilibria, appears to be a fruitful starting point for further research in this direction.

[31] Kurozumi (2008) follows a similar approach.

A Computation of the Private Sector's Response

As mentioned in the main text, (8) and (9) can be used to replace $\mathbb{E}_t[\pi_{t+1}]$ and $E_t[y_{t+1}]$ in (1) and (2). The resulting expressions

$$\pi_t = \beta\psi_m^\pi\left(\phi_m m_t + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t\right) + \kappa y_t + \xi_t, \tag{16}$$

$$\begin{aligned} y_t = &-\sigma\left(i_t - \psi_m^\pi\left(\phi_m m_t + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t\right)\right) \\ &+ \psi_m^y\left(\phi_m m_t + \phi_\pi \pi_t + \phi_y y_t + \phi_i i_t\right) + \mu_t \end{aligned} \tag{17}$$

can be combined to derive the private sector's choices of inflation, $\mathcal{P}(i_t, s_t)$, and the output gap, $\mathcal{Y}(i_t, s_t)$. It is straightforward but tedious to show that this procedure yields:

$$\begin{aligned} \mathcal{P}(i_t, s_t) = &\frac{1}{1 - \psi_m^\pi((\beta + \sigma\kappa)\phi_\pi + \sigma\phi_y) - \psi_m^y(\kappa\phi_\pi + \phi_y)} \\ &\times \Bigg[\left((\beta\phi_i + \kappa\sigma\phi_i - \beta\phi_y\sigma)\psi_m^\pi + \kappa\phi_i\psi_m^y - \kappa\sigma\right) i_t \\ &\quad + \phi_m\left((\beta + \sigma\kappa)\psi_m^\pi + \kappa\psi_m^y\right) m_t \\ &\quad + \left(\beta\phi_y\psi_m^\pi + \kappa\right)\mu_t + \left(1 - \sigma\phi_y\psi_m^\pi - \phi_y\psi_m^y\right)\xi_t\Bigg], \end{aligned} \tag{18}$$

and

$$\begin{aligned} \mathcal{Y}(i_t, s_t) = &\frac{1}{1 - \psi_m^\pi((\beta + \sigma\kappa)\phi_\pi + \sigma\phi_y) - \psi_m^y(\kappa\phi_\pi + \phi_y)} \\ &\times \Bigg[\left(\sigma(\phi_i + \beta\phi_\pi)\psi_m^\pi + \phi_i\psi_m^y - \sigma\right) i_t + \phi_m\left(\sigma\psi_m^\pi + \psi_m^y\right) m_t \\ &\quad + \left(1 - \beta\phi_\pi\psi_m^\pi\right)\mu_t + \phi_\pi\left(\sigma\psi_m^\pi + \psi_m^y\right)\xi_t\Bigg]. \end{aligned} \tag{19}$$

We note that only combinations of coefficients are admissible for which the denominator in (18) and (19) is not negative. Thus we obtain

$$\psi_m^\pi((\beta + \sigma\kappa)\phi_\pi + \sigma\phi_y) + \psi_m^y(\kappa\phi_\pi + \phi_y) \neq 1. \tag{20}$$

Expressions (18) and (19) are somewhat difficult to interpret in general. In the special case with $\psi_m^\pi = \psi_m^y = 0$, which corresponds to the standard discretionary equilibrium, we obtain

$$\mathcal{P}(i_t, s_t) = \kappa(-\sigma i_t + \mu_t) + \xi_t, \tag{21}$$

$$\mathcal{Y}(i_t, s_t) = -\sigma i_t + \mu_t. \tag{22}$$

In this special case, the second equation can be interpreted easily: It represents the IS curve for $\mathbb{E}_t[y_{t+1}] = 0$. The first equation is the Phillips curve with zero inflation expectations, wherein the output gap has been replaced by the expression from the IS equation. The responses of output and inflation are standard in this case: If the central bank raises interest rates, this lowers output according to the IS curve (22), and thereby also lowers inflation.

The general expressions (18) and (19) are significantly more complex because changes in interest rates not only have a direct effect on output and inflation as in (21) and (22), they also influence these variables through their impact on expectations about future inflation and output. $\square$

B Derivation of (11)

The first-order condition of (10) is

$$\pi_t \mathcal{P}_i + a y_t \mathcal{Y}_i + \beta \mathbb{E}_t[V_m(s_{t+1})] \left(\phi_\pi \mathcal{P}_i + \phi_y \mathcal{Y}_i + \phi_i\right) = 0, \tag{23}$$

where we have used that, in equilibrium, $\pi_t = \mathcal{P}(i_t, s_t)$ and $y_t = \mathcal{Y}(i_t, s_t)$. $V_m(s_{t+1})$ denotes the derivative of the value function with respect to m_{t+1}, and $\mathcal{P}_i$ and $\mathcal{Y}_i$ describe the partial derivatives of $\mathcal{P}$ and $\mathcal{Y}$ with respect to i_t. We observe that $\mathcal{P}_i$ and $\mathcal{Y}_i$ correspond to constants due to our assumption of stationarity (see (18) and (19)).

Applying the envelope theorem to (10) gives us the following expression for the value function's derivative with respect to m_t:

$$V_m(s_t) = \pi_t \mathcal{P}_m + a y_t \mathcal{Y}_m + \beta \mathbb{E}_t[V_m(s_{t+1})] \left(\phi_m + \phi_\pi \mathcal{P}_m + \phi_y \mathcal{Y}_m\right) \tag{24}$$

We shift Equation (24) one period forward, take expectations from period t, and use the resulting expression to substitute for $\mathbb{E}_t[V_m(s_{t+1})]$ in (23). This procedure gives

$$\begin{aligned}\pi_t\mathcal{P}_i + ay_t\mathcal{Y}_i + \beta\left(\phi_\pi\mathcal{P}_i + \phi_y\mathcal{Y}_i + \phi_i\right)\left(\mathbb{E}_t[\pi_{t+1}]\mathcal{P}_m + a\mathbb{E}_t[y_{t+1}]\mathcal{Y}_m\right)\\ + \beta^2\left(\phi_\pi\mathcal{P}_i + \phi_y\mathcal{Y}_i + \phi_i\right)\mathbb{E}_t[V_m(s_{t+2})]\left(\phi_m + \phi_\pi\mathcal{P}_m + \phi_y\mathcal{Y}_m\right) = 0.\end{aligned} \tag{25}$$

Shifting (23) one period forward and taking expectations from period t yields

$$\beta\mathbb{E}_t[V_m(s_{t+2})]\left(\phi_\pi\mathcal{P}_i + \phi_y\mathcal{Y}_i + \phi_i\right) = -\mathbb{E}_t[\pi_{t+1}]\mathcal{P}_i - a\mathbb{E}_t[y_{t+1}]\mathcal{Y}_i.$$

This expression can be used to rewrite (25) as

$$\begin{aligned}\pi_t\mathcal{P}_i + ay_t\mathcal{Y}_i + \beta\left(\phi_\pi\mathcal{P}_i + \phi_y\mathcal{Y}_i + \phi_i\right)\left(\mathbb{E}_t[\pi_{t+1}]\mathcal{P}_m + a\mathbb{E}_t[y_{t+1}]\mathcal{Y}_m\right)\\ - \beta\left(\phi_m + \phi_\pi\mathcal{P}_m + \phi_y\mathcal{Y}_m\right)\left(\mathbb{E}_t[\pi_{t+1}]\mathcal{P}_i + a\mathbb{E}_t[y_{t+1}]\mathcal{Y}_i\right) = 0.\end{aligned} \tag{26}$$

Using (18) and (19) to compute the derivatives of $\mathcal{P}$ and $\mathcal{Y}$, inserting these expressions into (26) and simplifying entails (11). □

C Proof of Proposition 1

We divide our analysis of arbitrary, non-standard MDEs into several steps. In our first step, we show that ψ_m^π can be normalized to one without loss of generality.

Normalization of $\boldsymbol{\psi_m^\pi}$ We begin our analysis by showing that, if $\psi_m^\pi = 0$, we will always arrive at the standard equilibrium. For this purpose, we note that (16) simplifies to

$$\pi_t = \kappa y_t + \xi_t.$$

Inserting π_t and y_t from (6) and (7) for $\psi_m^\pi = 0$ yields

$$\psi_\xi^\pi\xi_t + \psi_\mu^\pi\mu_t = \kappa\left(\psi_m^y m_t + \psi_\xi^y\xi_t + \psi_\mu^y\mu_t\right) + \xi_t.$$

This equation has to hold for arbitrary values of m_t. As a result, $\psi_m^y = 0$. We have already shown that $\psi_m^\pi = \psi_m^y = 0$ results in the standard equilibrium. Hence, we assume $\psi_m^\pi \neq 0$ in the following. According to Lemma 2, it is possible to normalize $\psi_m^\pi = 1$ for the remainder of the analysis. Next we will derive a system of equations that will determine the coefficients $(\psi_m^y, \phi_m, \phi_\pi, \phi_y, \phi_i, \psi_\xi^\pi, \psi_\mu^\pi, \psi_\xi^y, \psi_\mu^y)$.

Three restrictions on the coefficients, derived from the IS curve and the Phillips curve The first set of conditions on the coefficients mentioned above can be obtained by replacing $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[y_{t+1}]$ in (1) and (2) with the help of (8) and (9). This produces (16) and (17). Solving (17) for i_t and inserting into (16) yields an equation linear in π_t, y_t, μ_t, ξ_t, and m_t. Next we can replace π_t and y_t by (6) and (7) to obtain a homogeneous linear equation in the state variables μ_t, ξ_t, and m_t. Because the equation has to hold independently of the values of μ_t, ξ_t, and m_t, the three coefficients in front of these variables have to be zero. As a result, we obtain three conditions:

$$\begin{aligned}&\left(\sigma - \beta\psi_m^y - \kappa\psi_m^y\sigma - \kappa\left(\psi_m^y\right)^2 + \psi_m^y\right)\phi_i \\ &\qquad + \beta\sigma\phi_\pi + \beta\sigma\psi_m^y\phi_y - \sigma + \beta\phi_m\sigma + \kappa\psi_m^y\sigma = 0\end{aligned} \tag{27}$$

$$\begin{aligned}&\left(\psi_\xi^\pi\psi_m^y - \beta\psi_\xi^y - \kappa\psi_\xi^y\sigma + \psi_\xi^\pi\sigma - \sigma - \psi_m^y - \kappa\psi_\xi^y\psi_m^y\right)\phi_i \\ &\qquad + \beta\phi_y\psi_\xi^y\sigma - \sigma\psi_\xi^\pi + \beta\phi_\pi\psi_\xi^\pi\sigma + \kappa\psi_\xi^y\sigma + \sigma = 0\end{aligned} \tag{28}$$

$$\begin{aligned}&\left(\psi_\mu^\pi\sigma + \psi_\mu^\pi\psi_m^y + \beta - \kappa\psi_\mu^y\sigma - \kappa\psi_\mu^y\psi_m^y - \beta\psi_\mu^y\right)\phi_i \\ &\qquad + \beta\phi_y\psi_\mu^y\sigma + \kappa\psi_\mu^y\sigma - \sigma\psi_\mu^\pi + \beta\phi_\pi\psi_\mu^\pi\sigma = 0\end{aligned} \tag{29}$$

It will be useful for our future analysis that (27) does not depend on $(\psi_\xi^\pi, \psi_\mu^\pi, \psi_\xi^y, \psi_\mu^y)$. Moreover, (28) depends on $(\psi_\xi^\pi, \psi_\xi^y)$ but not on $(\psi_\mu^\pi, \psi_\mu^y)$, and (29) depends on $(\psi_\mu^\pi, \psi_\mu^y)$ but not on $(\psi_\xi^\pi, \psi_\xi^y)$.

Three additional conditions, derived from the equation describing optimal central bank behavior The condition guaranteeing optimal central bank behavior, (11), can be used to generate three additional restrictions on the coefficients. First, we replace $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[y_{t+1}]$, drawing on (8) and (9). As a result, we obtain an equation that is linear in π_t, y_t, i_t, μ_t, ξ_t, and m_t.

In the next step, we use (16) to replace i_t. Then we substitute for π_t and y_t with the help of (6) and (7). This produces a homogeneous, linear equation in the state variables μ_t, ξ_t, and m_t. As this condition has to hold independently of the values of μ_t, ξ_t, and m_t, the three coefficients in front of these variables must be zero. Consequently, we obtain three additional equations:

$$\begin{aligned}
&-\phi_i\kappa\psi_m^y - a\left(\psi_m^y\right)^2\phi_i - \phi_i\beta + a\psi_m^y\sigma - \phi_m\kappa\sigma \\
&\quad - a\psi_m^y\beta\phi_\pi\sigma - a\sigma\phi_m\psi_m^y + \beta\phi_y\sigma + \sigma\kappa^2\phi_m\psi_m^y - \phi_i\kappa\sigma \\
&\quad + a\phi_m\left(\psi_m^y\right)^2\kappa\sigma + \kappa\sigma - a\psi_m^y\sigma\phi_i = 0
\end{aligned} \tag{30}$$

$$\begin{aligned}
&-a\psi_\xi^y\sigma\phi_i - a\sigma\phi_m\psi_\xi^\pi\psi_m^y - \psi_\xi^\pi\kappa\psi_m^y\phi_i + \psi_\xi^\pi\beta\phi_y\sigma - a\psi_\xi^y\psi_m^y\phi_i \\
&\quad + \kappa^2\phi_m\sigma\psi_\xi^y - \psi_\xi^\pi\beta\phi_i - \psi_\xi^\pi\kappa\sigma\phi_i + a\psi_\xi^y\sigma + a\kappa\sigma\phi_m\psi_\xi^y\psi_m^y \\
&\quad - \sigma\phi_m\psi_\xi^\pi\kappa + \psi_\xi^\pi\kappa\sigma - a\psi_\xi^y\beta\phi_\pi\sigma + \sigma\phi_m\kappa + a\sigma\phi_m\psi_m^y = 0
\end{aligned} \tag{31}$$

$$\begin{aligned}
&-a\sigma\phi_m\psi_\mu^\pi\psi_m^y - \psi_\mu^\pi\kappa\psi_m^y\phi_i + \psi_\mu^\pi\beta\phi_y\sigma - a\psi_\mu^y\psi_m^y\phi_i + \kappa^2\phi_m\sigma\psi_\mu^y \\
&\quad - \psi_\mu^\pi\beta\phi_i - \psi_\mu^\pi\kappa\sigma\phi_i + a\psi_\mu^y\sigma + a\kappa\sigma\phi_m\psi_\mu^y\psi_m^y - \sigma\phi_m\psi_\mu^\pi\kappa \\
&\quad + \psi_\mu^\pi\kappa\sigma - a\psi_\mu^y\beta\phi_\pi\sigma - a\psi_\mu^y\sigma\phi_i = 0
\end{aligned} \tag{32}$$

Hence, according to this and the previous step, the coefficients $(\psi_\xi^\pi, \psi_\mu^\pi, \psi_m^y, \psi_\xi^y, \psi_\mu^y, \phi_m, \phi_\pi, \phi_y, \phi_i)$ have to satisfy (27)-(32).

Evaluation of (27) and (30) Out of these equations, (27) and (30) are special in that they do not depend on the coefficients associated with the shocks, i.e. ψ_ξ^π, ψ_ξ^y, ψ_μ^π, and ψ_μ^y. Moreover, they are linear in ϕ_π and ϕ_y. It is straightforward to verify that (27) and (30), interpreted as a linear system of equations in ϕ_π and ϕ_y are independent. Hence, for arbitrarily chosen $(\psi_m^y, \phi_m, \phi_i)$, (27) and (30) give unique solutions for ϕ_π and ϕ_y:

$$\tilde{\phi}_\pi = -\frac{\sigma + \psi_m^y}{\beta\sigma}\phi_i - \frac{\beta + \kappa\psi_m^y + a{\psi_m^y}^2 - a\kappa{\psi_m^y}^3 - \kappa^2{\psi_m^y}^2}{\beta\left(1 + a{\psi_m^y}^2\right)}\phi_m + \frac{1}{\beta} \tag{33}$$

$$\tilde{\phi}_y = \frac{\kappa\sigma + \beta + \kappa\psi_m^y}{\beta\sigma}\phi_i - \frac{a\psi_m^y\beta - \kappa + a\kappa{\psi_m^y}^2 - a\psi_m^y + \kappa^2\psi_m^y}{\beta\left(1 + a{\psi_m^y}^2\right)}\phi_m - \frac{\kappa}{\beta} \tag{34}$$

Ruling out explosive solutions In equilibrium, the evolution of π_t and y_t is given by (6) and (7), which specify π_t and y_t as linear functions of the state variables ξ_t, μ_t, and m_t. The requirements $\lim_{t\to\infty}\mathbb{E}_0[\pi_t] = 0$ and $\lim_{t\to\infty}\mathbb{E}_0[y_t] = 0$, which were introduced in the definition of MDE (see Definition 1), hold for arbitrary values of μ_0, ξ_0, and m_0 iff

$$\lim_{t\to\infty} \mathbb{E}_0[m_t] = 0 \tag{35}$$

holds for all μ_0, ξ_0, and m_0.

Hence we have to analyze the dynamic evolution of m_t. More specifically, we solve (16) for i_t and utilize the resulting expression to replace i_t in (4). Then we use (6) and (7) to substitute for π_t and y_t. As a result, we obtain an expression for m_{t+1} as a function of m_t and the shocks ξ_t and μ_t. Condition (35) holds if the coefficient in front of m_t in the expression for m_{t+1} lies in the interval $(-1,+1)$. It is straightforward to derive that this condition amounts to $\frac{1-\kappa\psi_m^y}{\beta} \in (-1,+1)$, which is equivalent to

$$\psi_m^y \in \left(\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}\right). \tag{36}$$

We obtain as a corollary that ψ_m^y is always positive. As we have normalized ψ_m^π to one, this implies that both inflation and output are shifted into the same direction by variations in m_t.

Evaluating (29) and (32) After we have analyzed (27) and (30) in detail, we need to look at the four remaining equations in (27)-(32) more closely. We observe that ψ_μ^π and ψ_μ^y show up only in (29) and (32) but not in (27), (28), (30), and (31).

For the moment, we focus on the case where $\phi_m \neq 0$. The case with $\phi_m = 0$ will be considered later. Using (33) and (34) to replace ϕ_π and ϕ_y in (29) and (32) and solving for ψ_μ^π and ψ_μ^y yields

$$\tilde{\psi}_\mu^\pi = \frac{\phi_i}{\sigma\phi_m}, \tag{37}$$

$$\tilde{\psi}_\mu^y = \frac{\psi_m^y\phi_i}{\sigma\phi_m}. \tag{38}$$

We note that our previous finding $\psi_m^y > 0$ entails that the signs of $\tilde{\psi}_\mu^\pi$ and $\tilde{\psi}_\mu^y$ are identical. Consequently, demand shocks μ_t push both inflation and output into the same direction on impact.

Evaluating (28) and (31) We observe that (28) and (31) are the only equations in (27)-(32) that depend on ψ_ξ^π and ψ_ξ^y. As these coefficients enter (28) and (31) linearly, we obtain unique solutions for ψ_ξ^π and ψ_ξ^y, given specific values of the other coefficients $(\psi_m^y, \phi_m, \phi_\pi, \phi_y, \phi_i)$.[32]

Ruling out knife-edge cases for which the private-sector responses are undefined At this stage, we have to recall that the coefficients must satisfy (20). This condition guarantees that the denominator in (18) and (19), i.e. in the expressions describing the private-sector responses $\mathcal{P}(i_t, s_t)$ and $\mathcal{Y}(i_t, s_t)$, is different from zero. Inserting (33) and (34) into (20) yields:

$$\begin{aligned} 0 \neq f(\psi_m^y) := a\kappa^2 {\psi_m^y}^4 &+ \kappa(a\kappa\sigma + a\beta + \kappa^2 - 2a){\psi_m^y}^3 \\ &+ (\kappa^3\sigma - 2a\kappa\sigma + \beta\kappa^2 - 2a\beta - 2\kappa^2 + a){\psi_m^y}^2 \\ &+ (a\sigma + \kappa - a\beta\sigma - 2\kappa^2\sigma - 2\beta\kappa)\psi_m^y - \beta\kappa\sigma - \beta^2 + \kappa\sigma \end{aligned} \tag{39}$$

We define $\mathbb{L}$ as the set containing all real roots of $f(\psi_m^y)$. As $f(\psi_m^y)$ is a polynomial of order four, $\mathbb{L}$ contains at most four elements. We have to make sure that $\psi_m^y \notin \mathbb{L}$.

The construction of equilibria with $\boldsymbol{\phi_m \neq 0}$ We combine the findings from our previous steps. We can find all equilibria with $\phi_m \neq 0$ by the following procedure:

1. Normalize $\psi_m^\pi = 1$.
2. Pick arbitrary $(\psi_m^y, \phi_m, \phi_i)$ with $\phi_m \neq 0$ and $\psi_m^y \in \left(\frac{1-\beta}{\kappa}, \frac{1+\beta}{\kappa}\right) \setminus \mathbb{L}$.
3. Use (33),(34), (37), and (38) to find the unique solutions for ϕ_π, ϕ_y, ψ_μ^π, and ψ_μ^y, given $(\psi_m^y, \phi_m, \phi_i)$.
4. Use (28) and (31) to determine the unique solutions for ψ_ξ^π and ψ_ξ^y.

[32]It is tedious but straightforward to show that (28) and (31), interpreted as linear equations of ψ_ξ^π and ψ_ξ^y, are independent.

The case $\phi_m = 0$ Finally, we need to consider the possibility that $\phi_m = 0$. For $\phi_m = 0$, inserting (33) and (34) into (29) yields $\phi_i = 0$. Together with $\phi_m = 0$, $\phi_i = 0$ entails that (33) and (34) simplify to

$$\tilde{\phi}_\pi = \frac{1}{\beta} \tag{40}$$

$$\tilde{\phi}_y = -\frac{\kappa}{\beta} \tag{41}$$

Inserting $\phi_m = 0$, $\phi_i = 0$, (40), and (41) into (28) leads to a contradiction. Hence no solutions exists in this case. □

D Proof of Proposition 2

As a first step, we derive expressions for the output gap y_t and inflation π_t as functions of m_0, $\{\mu_i\}_{i=0}^t$, $\{\xi_i\}_{i=0}^t$. These expressions can be obtained by noting that, in equilibrium, m_{t+1} can be expressed recursively as

$$m_{t+1} = c_m m_t + c_\xi \xi_t + c_\mu \mu_t, \tag{42}$$

where the coefficients c_m, c_ξ, and c_μ can be determined by combining (4), (6), (7), and (16). Combining (42) with (6) and (7) yields:

$$\pi_t = \psi_m^\pi (c_m)^t m_0 + \psi_\xi^\pi \xi_t + \psi_\mu^\pi \mu_t + \psi_m^\pi \sum_{i=0}^{t-1} (c_m)^{t-1-i} (c_\xi \xi_i + c_\mu \mu_i) \tag{43}$$

$$y_t = \psi_m^y (c_m)^t m_0 + \psi_\xi^y \xi_t + \psi_\mu^y \mu_t + \psi_m^y \sum_{i=0}^{t-1} (c_m)^{t-1-i} (c_\xi \xi_i + c_\mu \mu_i) \tag{44}$$

Two different combinations of parameters $(\psi_m^y, \phi_m, \phi_i), (\hat{\psi}_m^y, \hat{\phi}_m, \hat{\phi}_i) \in \mathbb{A}$ characterize identical equilibria iff all coefficients in (43) and (44) in front of the ξ's and μ's are the same in both cases. Equivalently, we have to examine whether different combinations of $(\psi_m^y, \phi_m, \phi_i), (\hat{\psi}_m^y, \hat{\phi}_m, \hat{\phi}_i) \in \mathbb{A}$ lead to different values of ψ_ξ^π, ψ_μ^π, ψ_ξ^y, ψ_μ^y, c_m, c_ξ, and c_μ.[33]

[33]Recall that we have normalized $\psi_m^\pi = 1$.

First, we focus on coefficients ψ_μ^π, ψ_μ^y, c_m, and c_μ. Recall $\psi_\mu^\pi = \frac{\phi_i}{\sigma\phi_m}$ (see (37)) and $\psi_\mu^y = \frac{\psi_m^y \phi_i}{\sigma\phi_m}$ (see (38)). When deriving condition (36), which excludes explosive dynamics, we have implicitly determined c_m as

$$c_m = \frac{1-\kappa\psi_m^y}{\beta}. \tag{45}$$

Moreover, it is straightforward to show

$$c_\mu = \frac{\phi_i}{\sigma\phi_m} \cdot \frac{1-\kappa\psi_m^y}{\beta} \tag{46}$$

by using (37) and (38). We note that only the ratio of ϕ_i and ϕ_m but not their levels influence the values of ψ_μ^π, ψ_μ^y, and c_μ. We arrive at the preliminary finding that different levels of ψ_m^y and ϕ_i/ϕ_m definitely induce different equilibria.

Second, we focus on coefficients ψ_ξ^π, ψ_ξ^y, and c_ξ. It is straightforward but very tedious to compute expressions for ψ_ξ^π, ψ_ξ^y, and c_ξ. These expressions, interpreted as functions of ϕ_i and ϕ_m are not homogeneous of degree zero. Together with our previous result that different values of ψ_m^y and ϕ_i/ϕ_m induce different values of c_m, ϕ_μ^π, and ψ_μ^y, we arrive at the conclusion that any two different $(\psi_m^y, \phi_m, \phi_i), (\hat{\psi}_m^y, \hat{\phi}_m, \hat{\phi}_i) \in \mathbb{A}$ characterize different equilibria. $\square$

E Proof of Proposition 3

According to (43), the impulse response of inflation to a cost-push shock $\xi_0 = 1$ for $t \geq 1$ is:

$$\pi_t = (c_m)^{t-1} c_\xi, \tag{47}$$

where we have used the normalization $\psi_m^\pi = 1$. Similarly, the impulse response in the case of a demand shock $\mu_0 = 1$ is

$$\pi_t = (c_m)^{t-1} c_\mu. \tag{48}$$

We have already shown that $c_m = \frac{1-\kappa\psi_m^y}{\beta}$ (see (45)). Thus, any value of $c_m \in [0,1)$ can be achieved by selecting the corresponding value of ψ_m^y. Together, these findings prove the proposition. $\square$

F Proof of Proposition 4

Welfare in an arbitrary MDE We use the unconditional expectation of (3) as a measure of welfare. For $t \to \infty$, (43) and (44) become

$$\pi_t = \psi_\xi^\pi \xi_t + \psi_\mu^\pi \mu_t + \psi_m^\pi \sum_{i=0}^{\infty} (c_m)^i \left(c_\xi \xi_{t-1-i} + c_\mu \mu_{t-1-i} \right), \tag{49}$$

$$y_t = \psi_\xi^y \xi_t + \psi_\mu^y \mu_t + \psi_m^y \sum_{i=0}^{\infty} (c_m)^i \left(c_\xi \xi_{t-1-i} + c_\mu \mu_{t-1-i} \right). \tag{50}$$

We are now in a position to derive formal expressions for the unconditional expectations of π_t^2 and y_t^2:

$$\mathbb{E}[\pi_t^2] = \left(\psi_\xi^\pi\right)^2 \sigma_\xi^2 + \left(\psi_\mu^\pi\right)^2 \sigma_\mu^2 + \frac{\left(c_\xi^2 \sigma_\xi^2 + c_\mu^2 \sigma_\mu^2\right) \left(\psi_m^\pi\right)^2}{1 - c_m^2}, \tag{51}$$

$$\mathbb{E}[y_t^2] = \left(\psi_\xi^y\right)^2 \sigma_\xi^2 + \left(\psi_\mu^y\right)^2 \sigma_\mu^2 + \frac{\left(c_\xi^2 \sigma_\xi^2 + c_\mu^2 \sigma_\mu^2\right) \left(\psi_m^y\right)^2}{1 - c_m^2} \tag{52}$$

Welfare in the standard discretionary equilibrium The unconditional variances of the output gap and inflation in the standard discretionary equilibrium can be obtained by evaluating (51) and (52) for the special case $\psi_\xi^\pi = a/(a + \kappa^2)$, $\psi_\xi^y = -\kappa/(a + \kappa^2)$, and $\psi_m^\pi = \psi_m^y = \psi_\mu^\pi = \psi_\mu^y = 0$:

$$\mathbb{E}[\pi_t^2] = \left(\psi_\xi^\pi\right)^2 \sigma_\xi^2 = \frac{a^2}{(a + \kappa^2)^2} \sigma_\xi^2, \tag{53}$$

$$\mathbb{E}[y_t^2] = \left(\psi_\xi^y\right)^2 \sigma_\xi^2 = \frac{\kappa^2}{(a + \kappa^2)^2} \sigma_\xi^2 \tag{54}$$

For the parameter values specified in Section 4, the unconditional expectation of per-period social losses is $\frac{1}{2}\mathbb{E}[\pi_t^2 + a y_t^2] = 0.1250 \sigma_\xi^2$.

Welfare for a timeless-perspective optimal commitment policy The timeless-perspective optimal commitment policy is the commitment policy that the central bank would have committed to a long time ago. It is well known (see Clarida et al. (1999), pp. 1703-1704) that it can be described by

$$\pi_t = \delta \pi_{t-1} + \delta(\xi_t - \xi_{t-1}), \tag{55}$$

$$y_t = \delta y_{t-1} - \frac{\kappa \delta}{\alpha} \xi_t, \tag{56}$$

where

$$\delta = \frac{1 - \sqrt{1 - 4\beta a^2}}{2a\beta}. \tag{57}$$

Iterating backward gives

$$\pi_t = \delta\xi_t - (1-\delta)\sum_{i=1}^{\infty}\delta^i\xi_{t-i}, \tag{58}$$

$$y_t = -\frac{\kappa\delta}{\alpha}\sum_{i=0}^{\infty}\delta^i\xi_{t-i}. \tag{59}$$

From these expressions, it is straightforward to compute the unconditional variances of inflation and output:

$$\mathbb{E}[\pi_t^2] = \frac{2\delta^2}{1+\delta}\sigma_\xi^2, \tag{60}$$

$$\mathbb{E}[y_t^2] = \frac{\kappa^2\delta^2}{\alpha^2}\frac{1}{1-\delta^2}\sigma_\xi^2. \tag{61}$$

The unconditional expectation of per-period social losses amounts to $0.1068\sigma_\xi^2$ for the parameters introduced in Section 4.

Socially Optimal MDEs To identify socially optimal MDEs, we use a grid search to find the combination of $(\psi_m^y, \phi_m, \phi_i) \in \mathbb{A}$ that results in the lowest possible welfare. We note that the respective per-period social losses are $0.1047\sigma_\xi^2$, which is lower than under optimal commitment ($0.1068\sigma_\xi^2$) and in the discretionary case ($0.1250\sigma_\xi^2$). □

G Proof of Proposition 5

According to (45), c_m can be chosen arbitrarily close to one by selecting a value of ψ_m^y smaller than but close to $\frac{1+\beta}{\kappa}$. Moreover, c_μ can be independently chosen to be a strictly positive number (see (46)). According to (51), this means that an MDE with arbitrarily high inflation variance can be found if $\sigma_\mu^2 > 0$. If $\sigma_\mu^2 = 0$ and $\sigma_\xi^2 > 0$, a similar argument can be applied. □

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